Test 1 Mechanics & Relativity 2018-2019

Thursday September 20, 2018, 9:00 – 11:00, Aletta Jacobshal

Before you start, read the following:
There are 3 problems for a total of 45 points
Write your name and student number on each sheet of paper
Make clear arguments and derivations and use correct notation
Support your arguments by clear drawings where appropriate
Write in a readable manner, illegible handwriting will not be graded

NAME: …………………………………..………………………………..……………………………
STUDENT NUMBER: s……………..……………………………..……………………………

Problem 1	**: ………. points out of 10**
Problem 2	**: ………. points out of 15**
Problem 3	**: ………. points out of 20**
Total	**: ………. points out of 45**

GRADE = 1 + #points/5 =

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STUDENT NUMBER: s……………..……………………………..……………………………

Problem 1 – Basics (10 points)

Indicate whether a statement is TRUE (T) or FALSE (F)
by placing an ✕ in the corresponding box: ☒.
You can make a correction by completely blacking out the wrong answer: ■
Score = #correct answers – 10 (minimum 0)

(a) Albert Einstein was the first to formulate the principle of relativity T: ☐ F :☒

(b) Written in SR units, the circumference of Earth is about 40,000km T: ☐ F :☒

(c) One inertial frame may be accelerating relative to another inertial frame T: ☐ F :☒

(d) Objects have the same kinetic energy in all inertial reference frames T: ☐ F :☒

(e) In Newtonian mechanics, coordinate time is frame-dependent T: ☐ F :☒

(f) The proper time between two events depends on the worldline of the clock T: ☒ F :☐

(g) Proper-time intervals are frame-independent T: ☒ F :☐

(h) Spacetime intervals may be frame-dependent T: ☐ F :☒

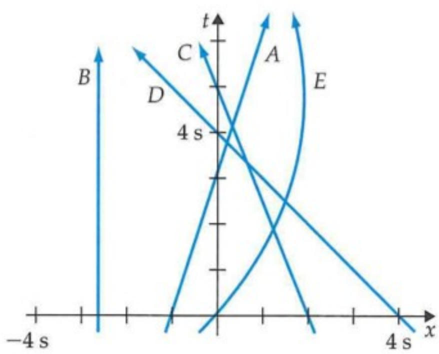
(i) The spacetime interval between two events can never be larger than the coordinate time

T: ☒ F :☐

(j) The worldline of a light ray emitted by a very fast moving object can have a slope larger than 1
T: ☐ F :☒

(k) The following frames are examples of (at least nearly) inertial reference frames:
- the frame attached to a non-rotating spaceship floating in deep space T: ☒ F :☐
- the frame attached to a car moving at constant velocity over a bumpy road T: ☐ F :☒
- the frame attached to a car accelerating over a smooth road T: ☐ F :☒
- the frame attached to the surface of Earth T: ☒ F :☐

(l) The spacetime diagram below shows the worldlines of several objects.
- all worldlines are possible T: ☒ F :☐
- object E is the only one accelerating T: ☒ F :☐
- object D could be light T: ☒ F :☐
- object C is moving faster than object A T: ☒ F :☐
- in a frame attached to object D objects A, B and C all move in the +x direction
T: ☒ F :☐
- except for object B, all object are always in motion T: ☐ F :☒



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Problem 2 – Collision course (15 points)

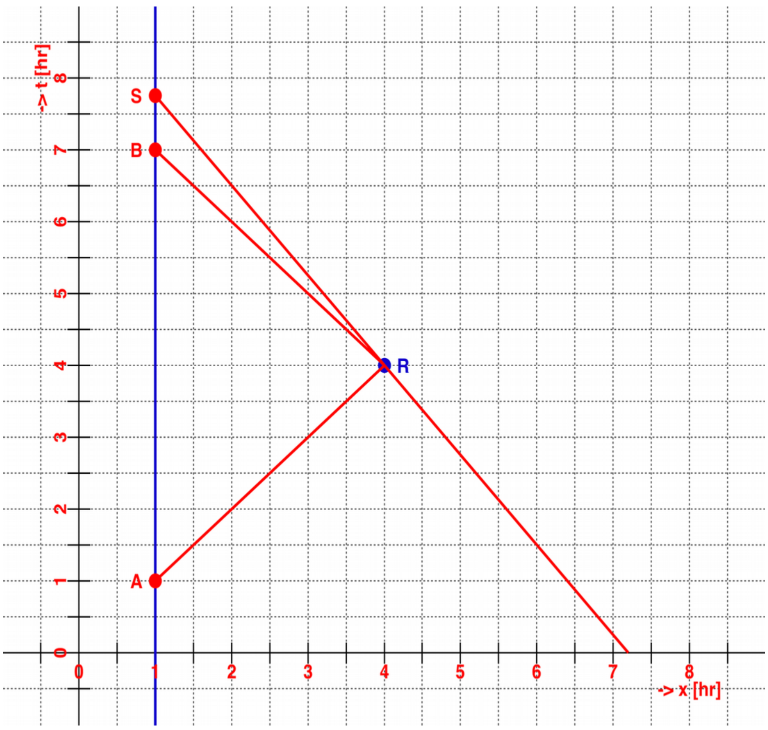
An astroid has been found to approach Earth at a constant speed of 0.8*c*. From Earth a radar signal is send out to detect it (event **A**), which reflects from the astroid (event **R**), and is again observed on Earth (event **B**). The coordinate time difference between events **A** and **B** is 6 hours. We want to find out when the asteroid hits Earth (**S**).

(a) In the spacetime diagram below the worldline of Earth and event **R** are shown. Make sure that the diagram can be read **unambiguously** by adding the necessary markings to the axes. (**6 points**) **[see figure; 1 point for each label, unit, numbers on axes]**

(b) Draw the worldlines of the asteroid and the radar signal, and indicate the events **A**, **B**, and **S**. (**6 points**) **[see figure; 1 points each of three worldline segments, A, B and S; worldline]**

(c) Using the figure, find the time Δt between detection of the radar reflection (**B**) and the moment the asteroid strikes Earth (**S**). (**3 points**)

Δt = 3¾hr – 3hr = ¾hr; to be read from axis **(½hr to 1hr are acceptable b/c precision)**



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Problem 3 – Unstable particles (20 points)

Muons are elementary particles that are created by cosmic rays in Earth's atmosphere at an altitude of 60km. Imagine that, after their creation, the muon travels straight downward towards Earth's surface. Each 2μs, as measured by the "internal clock" of the muons, 2/3 of the muons will have decayed, *i.e.* after 2μs 1/3 are left, after 4μs 1/3 of 1/3 = 1/9, after 6μs 1/3 x 1/3 x 1/3 = 1/27, *etc*. On average a muon will "live" for 2μs.

(a) What is the altitude (h) at which the muons are produced *in SR units*? (**3 points**)

h = 60km/300,000km/s = 200μs OR $2x10^{-4}$ s OR 0.2ms OR 0.0002s

(b) If special relativity were NOT true, fewer than 1 out of a 1000 muons would reach Earth (**3 points**) (**need to travel 200μs, while loosing 66% every 2 μs**) T: ☒ F :☐

(c) Give the definition of the spacetime interval Δs in terms of the (coordinate) time interval and the spatial distance between two events (*e.g.* creation and decay of a muon). (**3 points**)

$\Delta s = \sqrt{(\Delta t^2 - \Delta d^2)}$

(d) Calculate the spacetime interval between production and average decay *in the rest frame of a muon*, *i.e.* the reference frame in which the muon is at rest. (**3 points**)

$\Delta s_0 = \sqrt{[(2\mu s)^2 - 0^2]} = 2\mu s$ **because Δd=0**

(e) Calculate the spacetime interval between the production and decay of a muon *as observed from an inertial reference frame moving with velocity v*. Hint: first think, then calculate! (**3 points**)

$\Delta s_v = \Delta s_0 = 2\mu s$ **(invariance of spacetime interval)**

(f) If a muon is observed to decay just above the surface of Earth (h = 0), use Δt to argue that the muons have to move at a velocity v= Δh/Δt less than 1% below *c*. (**5 points**)

$\Delta s = 2\mu s = \sqrt{(\Delta t^2 - \Delta h^2)}$; $\Delta h = 200\mu s$, $\Delta t = \Delta h/v$; filling in we find

$2\mu s = \sqrt{((\Delta h/v)^2 - \Delta h^2)} = \sqrt{(1/v^2 - 1)}\ \Delta h = \sqrt{(1/v^2 - 1)}\ 200\mu s$

So $\sqrt{(1/v^2 - 1)} = 2\mu s/200\mu s = 0.01 \rightarrow 1/v^2 - 1 = 0.0001 \rightarrow 1/v^2 = 1.0001$

$\rightarrow v^2 = 1/1.0001 = 0.9999 \rightarrow v = 0.99995$

So v is 0.005% lower than c=1